AP PHYSICS 1: SUMMER ASSIGNMENT 2017

Dear Future AP Physics Student,

Here is your much-anticipated summer assignment. The purpose of this assignment is not to punish you for signing up for the course, but to help refresh your memory of some basic mathematical principles and get a jump-start on our upcoming challenge. Before you begin the summer assignment there are a few important points that must be addressed.

First, I am committed to helping you be successful in my class, however it is important to point out that the course is called AP Physics. This is a true college level course, ***not*** an Honors Course. You must be able to handle material at a college pace and be able to learn on your own from resources provided. The key to success in a college physics course is the desire to challenge oneself and the ability to persevere in a stressful academic environment.

Two, this course is intended for students who have completed both Honors Chemistry and Honors Algebra II / Trig. Of course you can still be successful if you have not completed these courses, but be prepared to work hard! **(If you are in Geometry you absolutely cannot take this class!)**

Three, the summer assignment is a review of basic mathematical principals as well as an introduction to Chapter 1 of our course. For this summer assignment you are asked to read all of Chapter 1 and then complete the assigned chapter problems (on a separate sheet of paper). You will find some parts of this assignment easy and some parts quite challenging. You will likely have trouble finishing the assignment, which is expected. *Do Your Best!* **The summer assignment is needed in class on the first day of school.**

I am looking forward to working with you this fall. Physics is a fun course, and I have a great year planned for us. Have a relaxing summer!

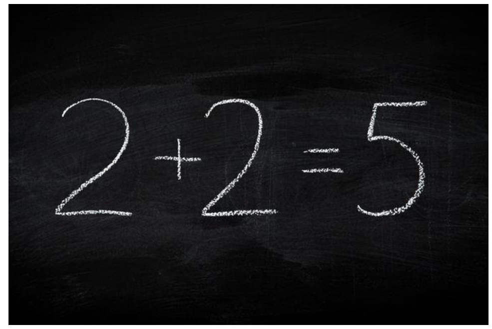
Mr. Sneider

For help understanding Chapter 1 refer to
https://sites.google.com/site/twuphysicslessons/home/kinematics/kinematics-page-4

You may find the videos titled “22”, “24”, and “25” to be particularly helpful. Good Luck!

Chapter 1
Introduction & Mathematical Concepts

Chapter 1 Assignment
Reading
Read 1.1 – 1.8



CHAPTER 1 PROBLEMS:

1.1 - 1.3	# 1a,b,c; 2, 4
1.4	# 11, 13, 15, 18, 19
1.6	# 22a,b; 23, 25a,b; 26a,b
1.7	# 32a,b; 35a,b; 36a,b
1.8	**# 43, 45, 47, 51a,b (We will discuss these)**

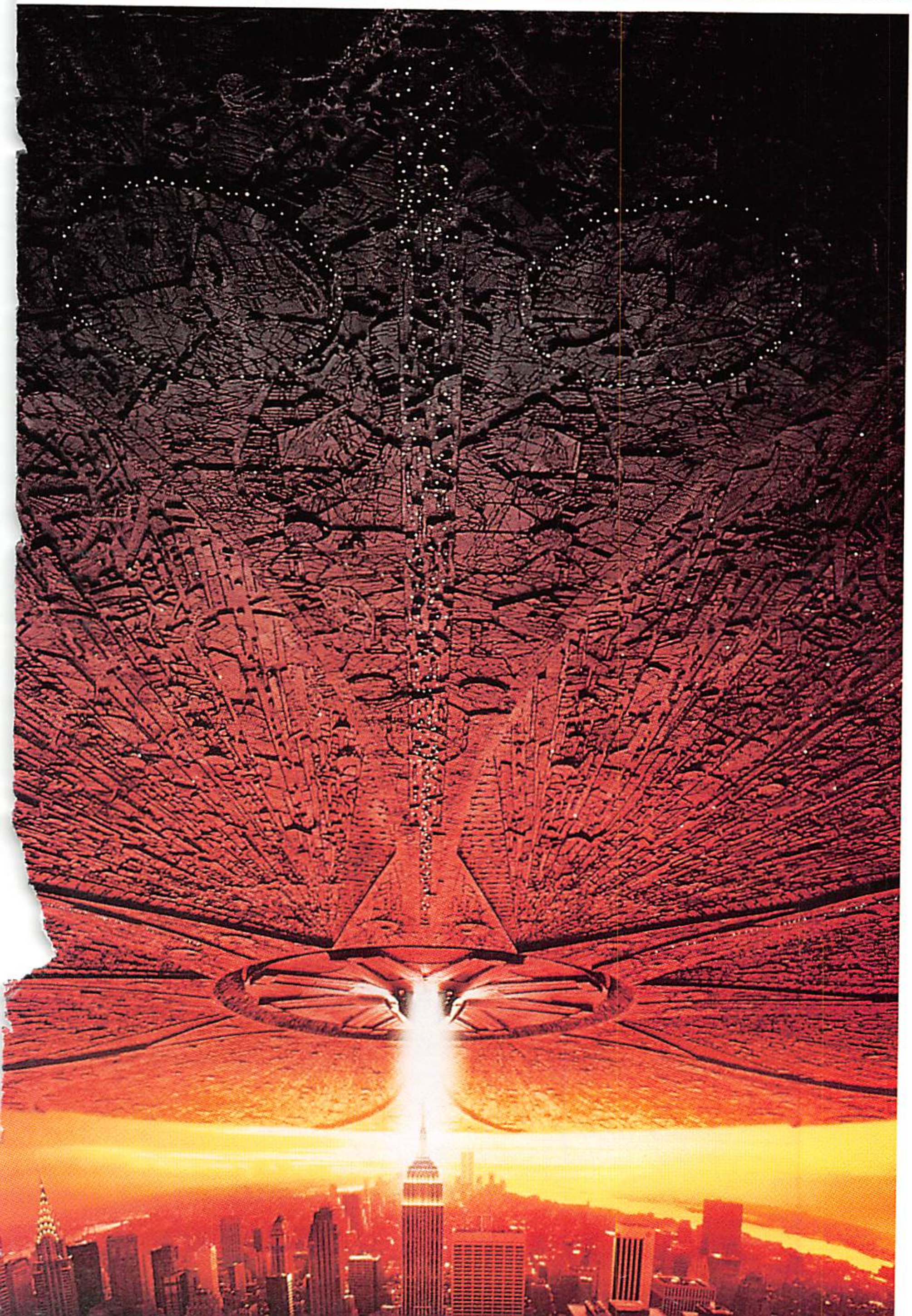
SOLUTIONS TO PROBLEMS:

#1	a. 5×10^{-3} g
	b. 5 mg
	c. 5×10^{-3} μg
#2	42,200 m
# 4.	$v = 840\ ^{km}/_{hr}$
# 11.	713 m
# 13.	80.1 km at 25.9^o south of west
# 15.	3.73 m
# 18.	c = 0.487 nm
# 19.	35.3^o
# 22	a. d = 64 m
	b. θ = 37° south of east
# 23.	200 N due east or 600 N due west
# 25.	a. 5.70×10^2 Newtons
	b. 33.6^o south of west
# 26.	5.75 km at 58.5 ^{o} west of south
# 32.	a. $r_x = 143$ m
	b. $r_y = -104$ m
# 35.	a. 147 km
	b. 47.9 km
# 36.	a. $F = 2.0 \times 10^2$ N
	b. θ = 41°
# 43.	268km at 38.5^o north of east
# 45.	7.1 m at 9.9^o north of east
# 47.	30.2 m at 10.2^o
# 51.	a. 371 units
	b. 354 units

C H A P T E R

1

INTRODUCTION AND MATHEMATICAL CONCEPTS



The movie *Independence Day* is a tour de force of animation techniques, which rely heavily on computers and mathematical concepts. This chapter introduces some of the mathematical concepts—like trigonometry and vectors—that will be useful throughout this book in dealing with the laws of physics.

1.1 THE NATURE OF PHYSICS

The science of physics has developed out of the efforts of men and women to explain why our physical environment behaves as it does. These efforts have been so successful that the laws of physics now encompass a remarkable variety of phenomena, from planets orbiting the sun to lasers being used in eye surgery.

The laws of physics are equally remarkable for their scope. They describe the behavior of particles many times smaller than an atom and objects many times larger than our sun. The same laws apply to the heat generated by a burning match and the heat generated by a rocket engine. The same laws guide an astronomer in using the light from a distant star to determine how fast the star is moving and a police officer in using radar to catch a speeder. Physics can be applied fruitfully to objects as different as subatomic particles, distant stars, or speeding automobiles because it focuses on issues that are truly basic to the way nature works.

The strength of physics derives from the fact that its laws are based on experiment. This is not to say that intuition and educated guesses are unimportant. The great creative geniuses in science, as in art and music, work in leaps and bounds that no one can fully understand. In physics, however, a flash of insight never becomes accepted law unless its implications can be verified by experiment. This insistence on experimental verification has enabled physicists to build a rational and coherent understanding of nature.

The exciting feature of physics is its capacity for predicting how nature will behave in one situation on the basis of experimental data obtained in another situation. Such predictions place physics at the heart of modern technology and, therefore, can have a tremendous impact on our lives. Rocketry and the development of space travel have their roots firmly planted in the physical laws of Galileo Galilei (1564–1642) and Isaac Newton (1642–1727). The transportation industry relies heavily on physics in the development of engines and the design of aerodynamic vehicles. Entire electronics and computer industries owe their existence to the invention of the transistor, which grew directly out of the laws of physics that describe the electrical behavior of solids. The telecommunications industry depends extensively on electromagnetic waves, whose existence was predicted by James Clerk Maxwell (1831–1879) in his theory of electricity and magnetism. The medical profession uses X-ray, ultrasonic, and magnetic resonance methods for obtaining images of the interior of the human body, and physics lies at the core of all these. Perhaps the most widespread impact in modern technology is that due to the laser. Fields ranging from space exploration to medicine benefit from this incredible device, which is a direct application of the principles of atomic physics.

Because physics is so fundamental, it is a required course for students in a wide range of major areas. We welcome you to the study of this fascinating topic. You will learn how to see the world through the "eyes" of physics and to reason as a physicist does. In the process, you will learn how to apply physics principles to a wide range of problems. We hope that you will come to recognize that physics has important things to say about your environment.

1.2 UNITS

DEFINITION OF STANDARD UNITS

Physics experiments involve the measurement of a variety of quantities, and a great deal of effort goes into making these measurements as accurate and reproducible as possible. The first step toward ensuring accuracy and reproducibility is defining the units in which the measurements are made.

In this text, we will stress the system of units known according to the French phrase "Le **S**ystème **I**nternational d'Unités," referred to simply as ***SI units.*** This system, by international agreement, employs the ***meter*** (m) as the unit of length, the ***kilogram*** (kg) as the unit of mass, and the ***second*** (s) as the unit of time. Two other systems of units are worth mentioning. The CGS system utilizes the centimeter (cm), the gram (g), and the second for length, mass, and time, respectively, whereas the BE or British Engineering system (the gravitational version) uses the foot (ft), the slug (sl), and the second. Table 1.1 summarizes the units used for length, mass, and time in the three systems.

Originally, the meter as a unit of length was defined in terms of the distance measured along the earth's surface between the north pole and the equator. Eventually, a more accurate measurement standard was needed, and by international agreement the meter became the distance between two marks on a bar of platinum-iridium alloy (see Figure 1.1) kept at a temperature of 0 °C. Today, to meet further demands for increased accuracy, the meter is defined as the distance that light travels in a vacuum in a time of 1/299 792 458 second. This definition arises because the speed of light is a universal constant that is defined to be 299 792 458 m/s.

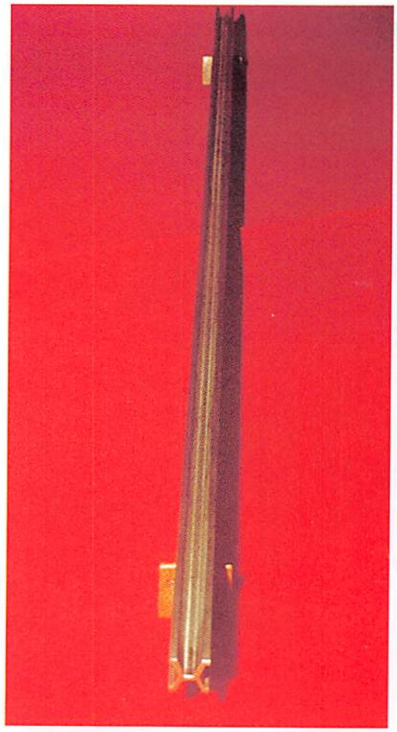


Figure 1.1 The standard platinum-iridium meter bar.

The definition of a kilogram as a unit of mass has also undergone changes over the years. As Chapter 4 discusses, the mass of an object indicates the tendency of the object to continue in motion with a constant velocity. Originally, the kilogram was expressed in terms of a specific amount of water. Today, one kilogram is defined to be the mass of a standard cylinder of platinum–iridium alloy, like the one in Figure 1.2.

As with the units for length and mass, the present definition of the second as a unit of time is different from the original definition. Originally, the second was defined according to the average time for the earth to rotate once about its axis, one day being set equal to 86 400 seconds. The earth's rotational motion was chosen because it is naturally repetitive, occurring over and over again. Today, we still use a naturally occurring repetitive phenomenon to define the second, but of a very different kind. We use the electromagnetic waves emitted by cesium-133 atoms in an

Figure 1.2 The standard platinum-iridium kilogram is kept at the International Bureau of Weights and Measures in Sèvres, France.

Table 1.1 Units of Measurement

	System		
	SI	CGS	BE
Length	meter (m)	centimeter (cm)	foot (ft)
Mass	kilogram (kg)	gram (g)	slug (sl)
Time	second (s)	second (s)	second (s)

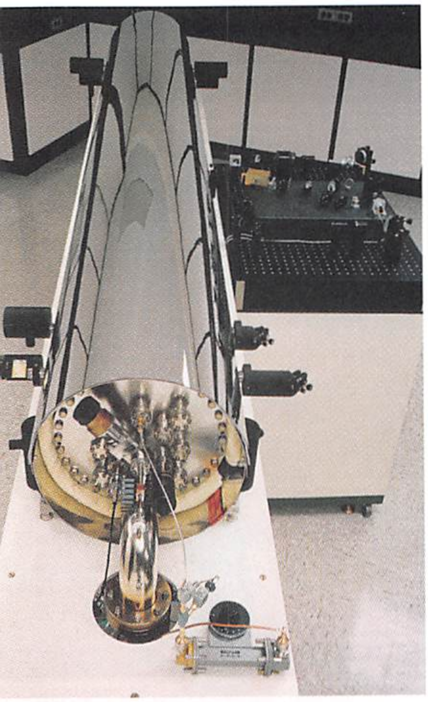


Figure 1.3 A cesium atomic clock.

atomic clock like that in Figure 1.3. One second is defined as the time needed for 9 192 631 770 wave cycles to occur.*

BASE UNITS AND DERIVED UNITS

The units for length, mass, and time, along with a few other units that will arise later, are regarded as ***base*** SI units. The word "base" refers to the fact that these units are used along with various laws to define additional units for other important physical quantities, such as force and energy. The units for these other physical quantities are referred to as ***derived*** units, since they are combinations of the base units. Derived units will be introduced as they arise naturally along with the related physical laws.

The value of a quantity in terms of base or derived units is sometimes a very large or very small number. In such cases, it is convenient to introduce larger or smaller units that are related to the normal units by multiples of ten. Table 1.2 summarizes the prefixes that are used to denote multiples of ten. For example, 1000 or 10^3 meters are referred to as 1 kilometer (km), and 0.001 or 10^{-3} meter is called 1 millimeter (mm). Similarly, 1000 grams and 0.001 gram are referred to as 1 kilogram (kg) and 1 milligram (mg), respectively. Appendix A contains a discussion of scientific notation and powers of ten, such as 10^3 and 10^{-3}.

Table 1.2 Standard Prefixes Used to Denote Multiples of Ten

Prefix	Symbol	Factor[a]
Tera	T	10^{12}
Giga[b]	G	10^{9}
Mega	M	10^{6}
Kilo	k	10^{3}
Hecto	h	10^{2}
Deka	da	10^{1}
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

[a] Appendix A contains a discussion of powers of ten and scientific notation.
[b] Pronounced jig'a.

1.3 THE ROLE OF UNITS IN PROBLEM SOLVING

THE CONVERSION OF UNITS

Since any quantity, such as length, can be measured in several different units, it is important to know how to convert from one unit to another. For instance, the foot can be used to express the distance between the two marks on the standard platinum-iridium meter bar. There are 3.281 feet in one meter, and this number can be used to convert from meters to feet, as the following example demonstrates.

EXAMPLE 1 • The World's Highest Waterfall

The highest waterfall in the world is Angel Falls in Venezuela, with a total drop of 979.0 m (see Figure 1.4). Express this drop in feet.

Reasoning When converting between units, we write down the units explicitly in the calculations and treat them like any algebraic quantity. In particular, we will take advantage of the following algebraic fact: Multiplying or dividing an equation by a factor of 1 does not alter the equation.

Solution Since 3.281 feet = 1 meter, it follows that (3.281 feet)/(1 meter) = 1. Using this factor of 1 to multiply the equation "Length = 979.0 meters," we find that

$$\text{Length} = (979.0\ \text{meters})(1) = (979.0\ \cancel{\text{meters}})\left(\frac{3.281\ \text{feet}}{1\ \cancel{\text{meter}}}\right) = \boxed{3212\ \text{feet}}$$

The colored lines emphasize that the units of meters behave like any algebraic quantity and cancel when the multiplication is performed, leaving only the desired unit of feet to describe the answer. In this regard, note that 3.281 feet = 1 meter also implies that (1 meter)/(3.281 feet) = 1. However, we chose not to multiply by a factor of 1 in this form, because the units of meters would not have canceled out.

* See Chapter 16 for a discussion of waves in general and Chapter 24 for a discussion of electromagnetic waves in particular.

A calculator gives the answer as 3212.099 feet. Standard procedures for significant figures, however, indicate that the answer should be rounded off to four significant figures, since the value of 979.0 meters is accurate to only four significant figures. In this regard, the "1 meter" in the denominator does not limit the significant figures of the answer, because this number is precisely one meter by definition of the conversion factor. Appendix B contains a review of significant figures.

Figure 1.4 Angel Falls in Venezuela is the highest waterfall in the world.

In any conversion, if the units do not combine algebraically to give the desired result, the conversion has not been carried out properly. The next example also stresses the importance of writing down the units and illustrates a typical situation in which several conversions are required.

EXAMPLE 2 • Interstate Speed Limit

Express the speed limit of 65 miles/hour in terms of meters/second.

Reasoning As in Example 1, it is important to write down the units explicitly in the calculations and treat them like any algebraic quantity. Here, two well-known relationships come into play, namely, 5280 feet = 1 mile and 3600 seconds = 1 hour. As a result, (5280 feet)/(1 mile) = 1 and (3600 seconds)/(1 hour) = 1. Multiplying and dividing by these factors of unity does not alter an equation, a fact that will aid us in the conversions.

Solution By multiplying and dividing by factors of unity, we can find the speed limit in feet per second as shown below:

$$\text{Speed} = \left(65\,\frac{\text{miles}}{\text{hour}}\right)(1)(1) = \left(65\,\frac{\cancel{\text{miles}}}{\cancel{\text{hour}}}\right)\left(\frac{5280\text{ feet}}{1\,\cancel{\text{mile}}}\right)\left(\frac{1\,\cancel{\text{hour}}}{3600\text{ s}}\right) = 95\,\frac{\text{feet}}{\text{second}}$$

To convert feet into meters, we use the fact that (1 meter)/(3.281 feet) = 1:

$$\text{Speed} = \left(95\,\frac{\text{feet}}{\text{second}}\right)(1) = \left(95\,\frac{\cancel{\text{feet}}}{\text{second}}\right)\left(\frac{1\text{ meter}}{3.281\,\cancel{\text{feet}}}\right) = \boxed{29\,\frac{\text{meters}}{\text{second}}}$$

A collection of useful conversion factors is given on the page facing the inside of the front cover. The reasoning strategy that we have followed in Examples 1 and 2 for converting between units is outlined as follows:

REASONING STRATEGY

Converting Between Units

1. In all calculations, write down the units explicitly.
2. Treat all units as algebraic quantities. In particular, when identical units are divided, they are eliminated algebraically.
3. Use the conversion factors located on the page facing the inside of the front cover. In your calculations, be guided by the fact that multiplying or dividing an equation by a factor of 1 does not alter the equation. For instance, the conversion factor of 3.281 feet = 1 meter might be applied in the form (3.281 feet)/(1 meter) = 1. This factor of 1 would be used to multiply an equation such as "Length = 5.00 meters" in order to convert meters to feet.
4. Check to see that your calculations are correct by verifying that the units combine algebraically to give the desired unit for the answer.

Units as a problem-solving aid

In addition to their role in guiding the use of conversion factors, units serve a useful purpose in solving problems. They can provide an internal check to eliminate certain kinds of errors, if they are carried along during each step of a calculation and treated like any algebraic factor.

Suppose, for instance, that the tank of a car contains 2.0 gallons of gas to start with and that gas is added at a rate of 7.0 gallons/minute. The total amount of gas in the tank 96 seconds later can be obtained by adding the amount put into the tank to the amount present initially. The amount put in can be calculated by multiplying the filling rate by the time the gas pump is on. But a lack of attention to the units in the calculation can lead to an erroneous result, as the following example shows.

$$\begin{aligned}\begin{matrix}\text{Total amount}\\\text{of gas}\end{matrix} &= \begin{matrix}\text{Gas initially}\\\text{present}\end{matrix} + \begin{matrix}\text{Gas}\\\text{added}\end{matrix}\\ &= 2.0\text{ gallons} + \left(7.0\,\frac{\text{gallons}}{\text{minute}}\right)(96\text{ seconds})\\ &= 2.0\text{ gallons} + 672\,\frac{\text{gallons}\cdot\text{seconds}}{\text{minute}}\end{aligned}$$

The answer cannot be $2.0 + 672 = 674$, because the units for the two added terms are not the same. ***Only quantities that have exactly the same units can be added (or subtracted).*** With the filling rate expressed as 7.0 gallons/minute, the correct answer can be obtained only if the time of 96 seconds is converted into minutes:

$$\text{Time} = (96\,\cancel{\text{seconds}})\left(\frac{1\text{ minute}}{60\,\cancel{\text{seconds}}}\right) = 1.6\text{ minutes}$$

$$\begin{aligned}\begin{matrix}\text{Total amount}\\\text{of gas}\end{matrix} &= 2.0\text{ gallons} + \left(7.0\,\frac{\text{gallons}}{\cancel{\text{minute}}}\right)(1.6\,\cancel{\text{minutes}})\\ &= 2.0\text{ gallons} + 11\text{ gallons} = 13\text{ gallons}\end{aligned}$$

As indicated by the colored lines, the units of time now cancel algebraically when the multiplication is carried out, leaving only the desired unit of gallons. The procedure of "carrying along the units" serves as an automatic reminder to convert all data used in a calculation into a consistent set of units.

This scientist is using an automatic pipette to deliver a fixed volume of liquid into a sample cell.

Dimensional analysis

We have seen that many quantities are denoted by specifying both a number and a unit. For example, the distance to the nearest telephone may be 8 meters, or the speed of a car might be 25 meters/second. Each quantity, according to its physical nature, requires a certain *type* of unit. Distance must be measured in a length unit such as meters, feet, or miles, and a time unit will not do. Likewise, the speed of an object must be specified as a length unit divided by a time unit. In physics, the term ***dimension*** is used to refer to the physical nature of a quantity and the type of unit used to specify it. Distance has the dimension of length (symbolized as [L]), while speed has the dimensions of length [L] divided by time [T], or [L/T]. Many physical quantities can be expressed in terms of a combination of fundamental dimensions such as length [L], time [T], and mass [M]. Later on, we will encounter certain other quantities, such as temperature, which are also fundamental. A fundamental quantity like temperature cannot be expressed as a combination of the dimensions of length, time, mass or any other fundamental dimension.

Dimensional analysis is used to check mathematical relations for the consistency of their dimensions. As an illustration, consider a car that starts from rest and accelerates to a speed v in a time t. Suppose we wish to calculate the distance x traveled by the car, but are not sure whether the correct relation is $x = \frac{1}{2}vt^2$ or $x = \frac{1}{2}vt$. We can decide by checking the quantities on both sides of the equals sign to see if they have the same dimensions. If the dimensions are not the same, the relation is incorrect. For $x = \frac{1}{2}vt^2$, we write the dimensions as follows, using the dimensions for distance [L], time [T], and speed [L/T]:

$$x = \tfrac{1}{2}vt^2$$

Dimensions:
$$[\mathrm{L}] \stackrel{?}{=} \left[\frac{\mathrm{L}}{\cancel{\mathrm{T}}}\right][\mathrm{T}]^{\cancel{2}} = [\mathrm{L}][\mathrm{T}]$$

Dimensions cancel just like algebraic quantities, and pure numerical factors like $\frac{1}{2}$ have no dimensions, so they can be ignored. The dimension on the left of the equals sign does not match those on the right, so the relation $x = \frac{1}{2}vt^2$ cannot be correct. On the other hand, applying dimensional analysis to $x = \frac{1}{2}vt$, we find that

$$x = \tfrac{1}{2}vt$$

Dimensions:
$$[\mathrm{L}] \stackrel{?}{=} \left[\frac{\mathrm{L}}{\cancel{\mathrm{T}}}\right][\cancel{\mathrm{T}}] = [\mathrm{L}]$$

• **PROBLEM SOLVING INSIGHT**
In problems that involve algebraic manipulations, you can check for errors that may have arisen during the manipulations by doing a dimensional analysis on the final expression.

The dimension on the left of the equals sign matches that on the right, so this relation is dimensionally correct. If we know that one of our two choices is the right one, then $x = \frac{1}{2}vt$ is it. In the absence of such knowledge, however, dimensional analysis cannot identify the correct relation. It can only identify which choices *may be* correct, since it does not account for numerical factors like $\frac{1}{2}$ or for the manner in which an equation was derived from physics principles.

1.4 TRIGONOMETRY

Scientists use mathematics to help them describe how the physical universe works, and trigonometry is an important branch of mathematics. Three trigonometric functions are utilized throughout this text. They are the sine, the cosine, and the tangent of the angle θ (Greek theta), abbreviated as $\sin\theta$, $\cos\theta$, and $\tan\theta$, respectively. These functions are defined below in terms of the symbols given along with the right triangle in Figure 1.5.

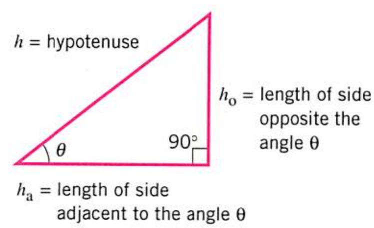


Figure 1.5 A right triangle.

■ DEFINITION OF SIN θ, COS θ, AND TAN θ

$$\sin\theta = \frac{h_o}{h} \tag{1.1}$$

$$\cos\theta = \frac{h_a}{h} \tag{1.2}$$

$$\tan\theta = \frac{h_o}{h_a} \tag{1.3}$$

h = length of the **hypotenuse** of a right triangle
h_o = length of the side **opposite** the angle θ
h_a = length of the side **adjacent** to the angle θ

The sine, cosine, and tangent of an angle are numbers without units, because each is expressed as the ratio of the lengths of two sides of a right triangle. Example 3 illustrates a typical application of Equation 1.3.

EXAMPLE 3 • Using Trigonometric Functions

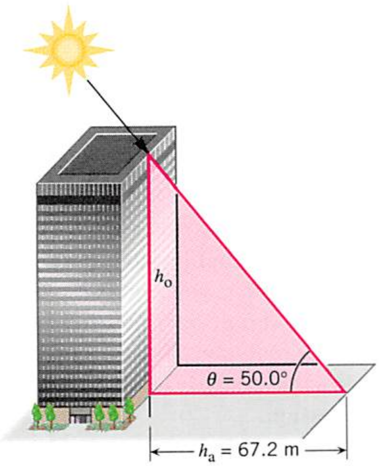


Figure 1.6 From a value for the angle θ and the length h_a of the shadow, the height h_o of the building can be found using trigonometry.

On a sunny day, a tall building casts a shadow that is 67.2 m long. The angle between the sun's rays and the ground is $\theta = 50.0°$, as Figure 1.6 shows. Determine the height of the building.

Reasoning Since we want to find the height of the building, we begin by identifying the height as the length h_o of the side opposite the angle θ in the colored right triangle in Figure 1.6. The length of the shadow is the length h_a of the side that is adjacent to the angle θ. The ratio of the length of the opposite side to the length of the adjacent side is the tangent of the angle θ, which can be used to find the height of the building.

Solution We use the tangent function in the following way, with $\theta = 50.0°$ and $h_a = 67.2$ m:

$$\tan\theta = \frac{h_o}{h_a} \tag{1.3}$$

$$h_o = h_a \tan\theta = (67.2 \text{ m})(\tan 50.0°) = (67.2 \text{ m})(1.19) = \boxed{80.0 \text{ m}}$$

The value of tan 50.0° is found by using a calculator.

The sine, cosine, or tangent may be used in calculations such as that in Example 3, depending on which side of the triangle has a known value and which side is asked for. However, ***the choice of which side of the triangle to label h_o (opposite) and which to label h_a (adjacent) can be made only after the angle θ is identified.***

Often the values for two sides of the right triangle in Figure 1.5 are available, and the value of the angle θ is unknown. The concept of ***inverse trigonometric functions*** plays an important role in such situations. Equations 1.4–1.6 give the inverse sine, inverse cosine, and inverse tangent in terms of the symbols used in the drawing. For instance, Equation 1.4 is read as "θ equals the angle whose sine is h_o/h."

$$\theta = \sin^{-1}\left(\frac{h_o}{h}\right) \tag{1.4}$$

$$\theta = \cos^{-1}\left(\frac{h_a}{h}\right) \tag{1.5}$$

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) \tag{1.6}$$

The use of "-1" as an exponent in Equations 1.4–1.6 *does not mean* "take the reciprocal." For instance, $\tan^{-1}(h_o/h_a)$ does not equal $1/\tan(h_o/h_a)$. Another way to express the inverse trigonometric functions is to use arc sin, arc cos, and arc tan instead of $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$. Example 4 illustrates the use of an inverse trigonometric function.

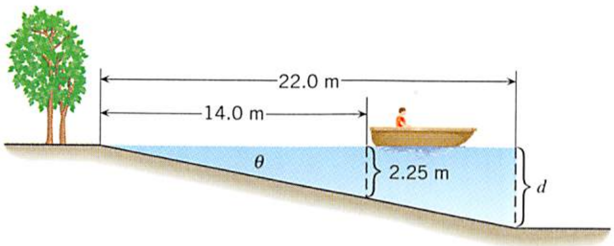


Figure 1.7 If the distance from the shore and the depth of the water at any one point are known, the angle θ can be found with the aid of trigonometry. Knowing the value of θ is useful, because then the depth d at another point can be determined.

EXAMPLE 4 • Using Inverse Trigonometric Functions

A lakefront drops off gradually at an angle θ, as Figure 1.7 indicates. For safety reasons, it is necessary to know how deep the lake is at various distances from the shore. To provide some information about the depth, a lifeguard rows straight out from the shore a distance of 14.0 m and drops a weighted fishing line. By measuring the length of the line, the lifeguard determines the depth to be 2.25 m. (a) What is the value of θ? (b) What would be the depth d of the lake at a distance of 22.0 m from the shore?

Reasoning Near the shore, the lengths of the opposite and adjacent sides of the right triangle in Figure 1.7 are $h_o = 2.25$ m and $h_a = 14.0$ m, relative to the angle θ. Having made this identification, we can use $\tan\theta = h_o/h_a$ to find the angle in part (a). For part (b) the procedure is similar, only farther from the shore the lengths of the opposite and adjacent sides become $h_o = d$ and $h_a = 22.0$ m. With the value for θ obtained in part (a), the tangent function can be used to find the unknown depth. Considering the way in which the lake bottom drops off in Figure 1.7, we expect the unknown depth to be greater than the value of 2.25 m that applies nearer the shore.

Solution

(a) Using Equation 1.3, we find that

$$\tan\theta = \frac{h_o}{h_a} = \frac{2.25\text{ m}}{14.0\text{ m}} = 0.161$$

Now that the value of $\tan\theta$ is known, the angle θ can be obtained by using the inverse tangent:

$$\theta = \tan^{-1}(0.161) = \boxed{9.15^\circ}$$

(b) With $\theta = 9.15°$, the tangent function can be used to find the unknown depth farther from the shore, where $h_o = d$ and $h_a = 22.0$ m. Since $\tan\theta = h_o/h_a$, it follows that

$$h_o = h_a \tan\theta$$
$$d = (22.0\text{ m})(\tan 9.15^\circ) = \boxed{3.54\text{ m}}$$

which is greater than 2.25 m, as expected.

The right triangle in Figure 1.5 provides the basis for defining the various trigonometric functions according to Equations 1.1–1.3. These functions always involve an angle and two sides of the triangle. There is also a relationship among the lengths of the three sides of a right triangle. This relationship is known as the ***Pythagorean theorem*** and is used often in this text.

PYTHAGOREAN THEOREM

The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides:

$$h^2 = h_o^2 + h_a^2 \quad (1.7)$$

1.5 THE NATURE OF PHYSICAL QUANTITIES: SCALARS AND VECTORS

SCALARS

The volume of water in a swimming pool might be 50 cubic meters, or the winning time of a race could be 11.3 seconds. In cases like these, only the size of the numbers matters. In other words, *how much* volume or time is there? The "50" specifies the amount of water in units of cubic meters, while the "11.3" specifies the amount of time in seconds. Volume and time are examples of scalar quantities. A ***scalar quantity*** is one that can be described by a single number (including any units) giving its size or magnitude. Some other common scalars are temperature (e.g., 20 °C) and mass (e.g., 85 kg).

VECTORS

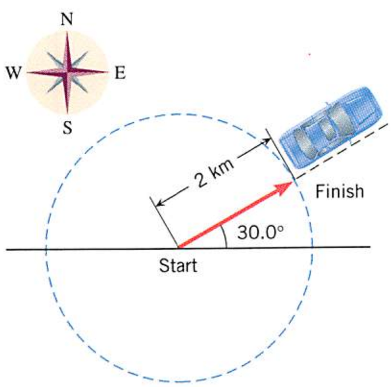


Figure 1.8 A vector quantity has a magnitude and a direction. The arrow in this drawing represents a displacement vector.

While many quantities in physics are scalars, there are also many that are not scalars, quantities for which magnitude tells only part of the story. Consider Figure 1.8, which depicts a car that has moved 2 km along a straight line from start to finish. When describing the motion, it is incomplete to say that "the car moved a distance of 2 km." This statement would indicate only that the car ends up somewhere on a circle whose center is at the starting point and whose radius is 2 km. A complete description must include the direction along with the distance, as in the statement "the car moved a distance of 2 km in a direction 30° north of east." A quantity that deals inherently with both magnitude and direction is called a ***vector quantity.*** Because direction is an important characteristic of vectors, arrows are used to represent them; ***the direction of the arrow gives the direction of the vector.*** The colored arrow in Figure 1.8, for example, is called the displacement vector, because it shows how the car is displaced from its starting point. Chapter 2 discusses this particular vector.

The length of the arrow in Figure 1.8 represents the magnitude of the displacement vector. If the car had moved 4 km instead of 2 km from the starting point, the arrow would have been drawn twice as long. ***By convention, the length of a vector arrow is proportional to the magnitude of the vector.***

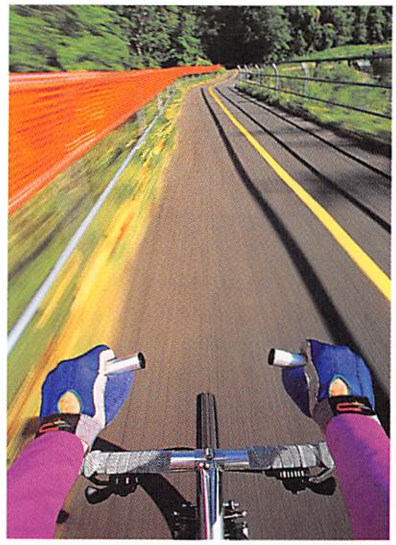
The practice of using the length of an arrow to represent the magnitude of a vector applies to any kind of vector. And in physics there are many important vectors, in addition to the displacement vector. All forces, for instance, are vectors. In common usage a force is a push or a pull, and the direction in which a force acts is just as important as the strength or magnitude of the force. The magnitude of a force is measured in SI units called newtons (N). An arrow representing a force of 20 newtons is drawn twice as long as one representing a force of 10 newtons.

The fundamental distinction between scalars and vectors is the characteristic of direction. Vectors have it, and scalars do not. Conceptual Example 5 helps to clarify this distinction and explains what is meant by the "direction" of a vector.

CONCEPTUAL EXAMPLE 5 • Vectors, Scalars, and the Role of Plus and Minus Signs

There are places where the temperature is +20 °C at one time of the year and −20 °C at another time. Do the plus and minus signs that signify positive and negative temperatures imply that temperature is a vector quantity?

Reasoning and Solution A vector has a physical direction associated with it, due east or due west, for example. The question, then, is whether such a direction is associated with temperature. In particular, do the plus and minus signs that go along with temperature imply this kind of direction? On a thermometer, the algebraic signs simply mean that the temperature is a number less than or greater than zero on the scale and have nothing to do with east, west, or any other physical direction. Temperature, then, is not a vector. It is a scalar, and scalars can sometimes be negative. ***The fact that a quantity is positive or negative does not necessarily mean that the quantity is a scalar or a vector.***



The velocity of this cyclist is another example of a vector quantity.

SYMBOLS USED FOR SCALARS AND VECTORS

Often, for the sake of convenience, quantities such as volume, time, displacement, and force are represented by symbols. This text follows the usual practice of writing vectors in boldface symbols* **(this is boldface)** and writing scalars in italic symbols (*this is italic*). Thus, a displacement vector is written as "**A** = 750 m, due east," where the **A** is a boldface symbol. By itself, however, separated from the direction, the magnitude of this vector is a scalar quantity. Therefore, the magnitude is written as "$A = 750$ m," where the A is an italic symbol.

1.6 VECTOR ADDITION AND SUBTRACTION

ADDITION OF COLINEAR VECTORS

Often it is necessary to add one vector to another, and the process of addition must take into account both the magnitude and the direction of the vectors. The simplest situation occurs when the vectors point along the same direction, that is, when they are colinear, as in Figure 1.9. Here, a car first moves along a straight line, with a displacement vector **A** of 275 m, due east. Then, the car moves again in the same direction, with a displacement vector **B** of 125 m, due east. These two vectors add to give the total displacement vector **R**, which would apply if the car had moved from start to finish in one step. The symbol **R** is used because the total vector is often called the ***resultant vector.*** With the tail of the second arrow located at the head of the first arrow, the two lengths simply add to give the length of the total displacement. This kind of vector addition is identical to the familiar addition of two scalar numbers (2 + 3 = 5), *and can be carried out here only because the vectors point along the same direction.* In such cases we add the individual magnitudes to get the magnitude of the total, knowing in advance what the direction must be. Formally, the addition is written as follows:

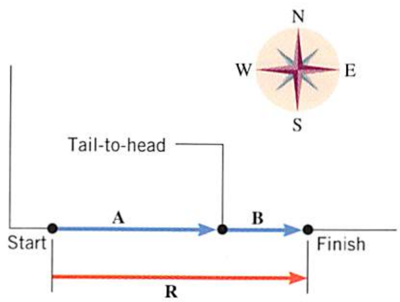


Figure 1.9 Two colinear displacement vectors **A** and **B** add to give the resultant displacement vector **R**.

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$\begin{aligned}\mathbf{R} &= 275 \text{ m, due east} + 125 \text{ m, due east}\\ &= 400 \text{ m, due east}\end{aligned}$$

* A vector quantity can also be represented without boldface symbols, by including an arrow above the symbol, e.g., $\vec{A}$.

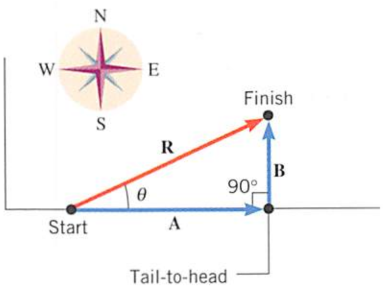


Figure 1.10 The addition of two perpendicular displacement vectors **A** and **B** gives the resultant vector **R**.

ADDITION OF PERPENDICULAR VECTORS

Perpendicular vectors are frequently encountered, and Figure 1.10 indicates how they can be added. This figure applies to a car that first travels with a displacement vector **A** of 275 m, due east, and then with a displacement vector **B** of 125 m, due north. The two vectors add to give a resultant displacement vector **R**. Once again, the vectors to be added are arranged in a tail-to-head fashion, and the resultant vector points from the tail of the first to the head of the last vector added. The resultant displacement is given by the vector equation

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

The addition in this equation cannot be carried out by writing $R = 275\text{ m} + 125\text{ m}$, because the vectors have different directions. Instead, we take advantage of the fact that the triangle in Figure 1.10 is a right triangle and use the Pythagorean theorem (Equation 1.7). According to this theorem, the magnitude of **R** is

$$R = \sqrt{(275\text{ m})^2 + (125\text{ m})^2} = 302\text{ m}$$

The angle θ in Figure 1.10 gives the direction of the resultant vector. Since the lengths of all three sides of the right triangle are now known, either $\sin\theta$, $\cos\theta$, or $\tan\theta$ can be used to determine θ:

$$\tan\theta = \frac{B}{A} = \frac{125\text{ m}}{275\text{ m}} = 0.455$$

$$\theta = \tan^{-1}(0.455) = 24.5°$$

Thus, the resultant displacement of the car has a magnitude of 302 m and points north of east at an angle of 24.5°. This displacement would bring the car from the start to the finish in Figure 1.10 in a single straight-line step.

ADDITION OF VECTORS THAT ARE NEITHER COLINEAR NOR PERPENDICULAR

When two vectors to be added are not perpendicular, the tail-to-head arrangement does not lead to a right triangle, and the Pythagorean theorem cannot be used. Figure 1.11*a* illustrates such a case for a car that moves with a displacement **A** of 275 m, due east and then with a displacement **B** of 125 m in a direction 55.0° north of west. As usual, the resultant displacement vector **R** is directed from the tail of the first to the head of the last vector added. The vector addition is still given according to

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

However, since the triangle in the drawing is not a right triangle, some means other than the Pythagorean theorem must be used to find the magnitude and direction of the resultant vector.

One approach uses a graphical technique. In this method, a diagram is constructed in which the arrows are drawn tail to head. The lengths of the vector arrows are drawn to scale, and the angles are drawn accurately (with a protractor, perhaps). Then, the length of the arrow representing the resultant vector is measured with a ruler. This length is converted into the magnitude of the resultant vector by using the scale factor with which the drawing is constructed. In Figure 1.11*b*, for example, a scale of one centimeter of arrow length for each 10.0 m of displacement is used, and it can be seen that the length of the arrow representing **R** is 22.8 cm. Since each centimeter corresponds to 10.0 m of displacement, the magnitude of **R**

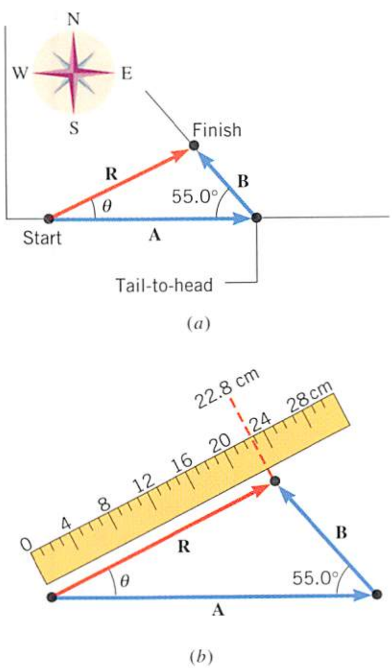


Figure 1.11 (*a*) The two displacement vectors **A** and **B** are neither colinear nor perpendicular but add to give the resultant vector **R**. (*b*) In one method for adding them together, a graphical technique is used.

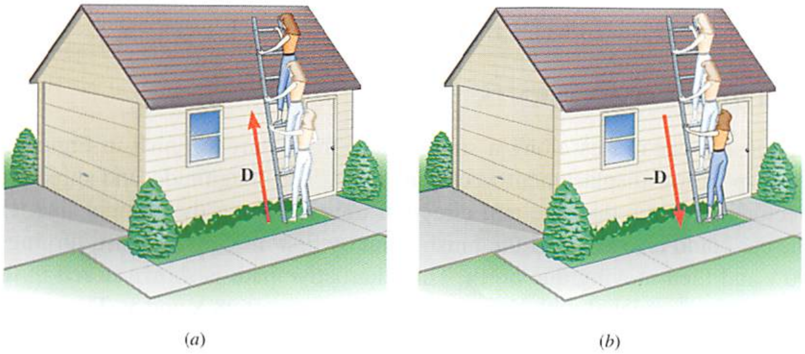


Figure 1.12 (*a*) The displacement vector for a woman climbing 1.2 m up a ladder is **D**. (*b*) The displacement vector for a woman climbing 1.2 m down a ladder is **−D**.

is 228 m. The angle θ, which gives the direction of **R**, can be measured with a protractor to be $\theta = 26.7°$.

SUBTRACTION OF VECTORS

The subtraction of one vector from another is carried out in a way that depends on the following fact. *When a vector is multiplied by −1, the magnitude of the vector remains the same, but the direction of the vector is reversed.* Conceptual Example 6 illustrates the meaning of this statement.

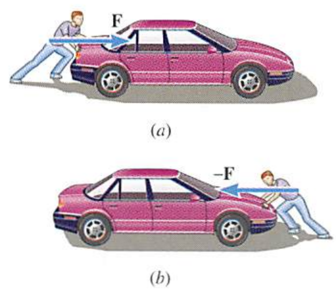


Figure 1.13 (*a*) The force vector for a man pushing on a car with 450 N of force in a direction due east is **F**. (*b*) The force vector for a man pushing on a car with 450 N of force in a direction due west is **−F**.

CONCEPTUAL EXAMPLE 6 • Multiplying a Vector by −1

Consider the two vectors described below:

1. A woman climbs 1.2 m up a ladder, so that her displacement vector **D** is 1.2 m, upward along the ladder, as in Figure 1.12*a*.
2. A man is pushing with 450 N of force on his stalled car, trying to move it eastward. The force vector **F** that he applies to the car is 450 N, due east, as in Figure 1.13*a*.

What are the physical meanings of the vectors **−D** and **−F**?

Reasoning and Solution A displacement vector of **−D** is (−1)**D** and has the same magnitude as the vector **D**, but is opposite in direction. Thus, **−D** would represent the displacement of a woman climbing 1.2 m down the ladder, as in Figure 1.12*b*. Similarly, a force vector of **−F** has the same magnitude as the vector **F**, but has the opposite direction. As a result, **−F** would represent a force of 450 N applied to the car in a direction of due west instead of due east, as in Figure 1.13*b*.

Related Homework Material: *Question 15, Problem 66*

In practice, vector subtraction is carried out exactly as vector addition, except that one of the vectors added is multiplied by a scalar factor of −1. To see why, look in Figure 1.14*a* at the two vectors **A** and **B**. These vectors add together to give a third vector **C**, according to **C = A + B**. Therefore, we can calculate vector **A** as **A = C − B**, which is an example of vector subtraction. However, we can also write this result as **A = C + (−B)** and treat it as vector addition. Figure 1.14*b* shows how to calculate vector **A** by adding the vectors **C** and **−B**. Notice that vectors **C** and **−B** are arranged tail to head and that any suitable method of vector addition can be employed to determine **A**.

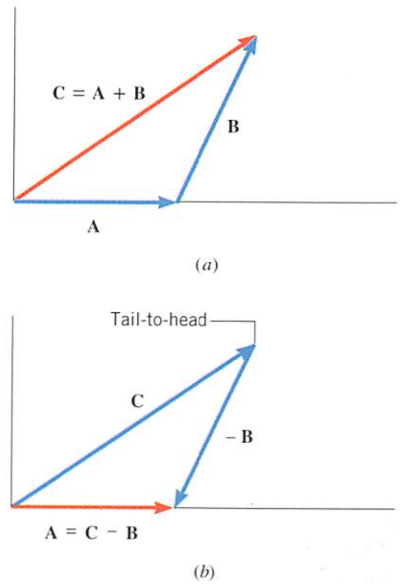


Figure 1.14 (*a*) Vector addition according to **C = A + B**. (*b*) Vector subtraction according to **A = C − B = C + (−B)**.

1.7 THE COMPONENTS OF A VECTOR

VECTOR COMPONENTS

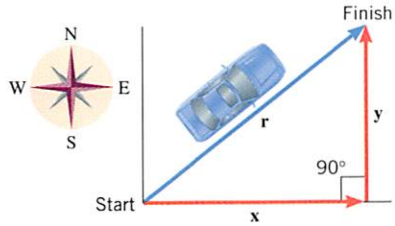


Figure 1.15 The displacement vector **r** and its vector components $\mathbf{x}$ and $\mathbf{y}$.

Suppose a car moves along a straight line from start to finish in Figure 1.15, the corresponding displacement vector being **r**. The magnitude and direction of the vector **r** give the distance and direction traveled along the straight line. However, the car could also arrive at the finish point by first moving due east, turning through 90°, and then moving due north. This alternative path is shown in red in the drawing and is associated with the two displacement vectors **x** and **y**. The vectors **x** and **y** are called the *x* vector component and the *y* vector component of **r**.

Vector components are very important in physics, and two basic features of them are apparent in Figure 1.15. One is that the components add together to equal the original vector, as expressed by the following vector equation:

$$\mathbf{r} = \mathbf{x} + \mathbf{y}$$

The components **x** and **y**, when added vectorially, convey exactly the same meaning as does the original vector **r**, that is, they indicate how the finish point is displaced relative to the starting point. In general, ***the components of any vector can be used in place of the vector itself in any calculation where it is convenient to do so.*** The other feature of vector components that is apparent in Figure 1.15 is that **x** and **y** are not just any two vectors that add together to give the original vector **r**; they are perpendicular vectors.* This perpendicularity is a valuable characteristic, as we will soon see.

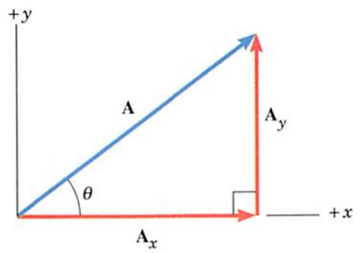


Figure 1.16 An arbitrary vector **A** and its vector components $\mathbf{A}_x$ and $\mathbf{A}_y$.

Any type of vector may be expressed in terms of its components, in a way similar to that illustrated for the displacement vector in Figure 1.15. Figure 1.16 shows an arbitrary vector **A** and its vector components $\mathbf{A}_x$ and $\mathbf{A}_y$. The components are drawn parallel to convenient *x* and *y* axes and are perpendicular. They add vectorially to equal the original vector **A**:

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$$

There are times when a drawing such as Figure 1.16 is not the most convenient way to represent vector components, and Figure 1.17 presents an alternative method. The disadvantage of this alternative is that the tail-to-head arrangement of $\mathbf{A}_x$ and $\mathbf{A}_y$ is missing, an arrangement that is a nice reminder that $\mathbf{A}_x$ and $\mathbf{A}_y$ add together to equal **A**.

The definition given below summarizes the meaning of vector components:

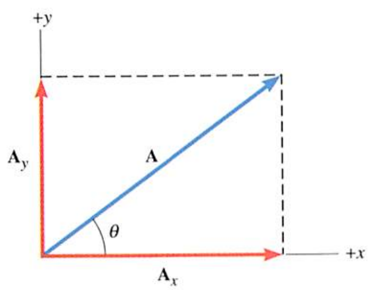


Figure 1.17 This alternative way of drawing the vector **A** and its vector components is completely equivalent to that shown in Figure 1.16.

■ **DEFINITION OF VECTOR COMPONENTS**

In two dimensions, the vector components of a vector **A** are two perpendicular vectors $\mathbf{A}_x$ and $\mathbf{A}_y$ that are parallel to the *x* and *y* axes, respectively, and add together vectorially so that $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$.

The values calculated for vector components depend on the orientation of the vector relative to the axes used as a reference. Figure 1.18 illustrates this fact for a vector **A**, by showing two sets of axes, one set being rotated clockwise relative to the other. With respect to the black axes, vector **A** has perpendicular vector components $\mathbf{A}_x$ and $\mathbf{A}_y$; with respect to the colored rotated axes, vector **A** has different vector

* It is possible to introduce vector components that are not perpendicular, but, in general, they are not as useful as those introduced here.

components $\mathbf{A}_x'$ and $\mathbf{A}_y'$. The choice of which set of components to use is purely a matter of convenience.

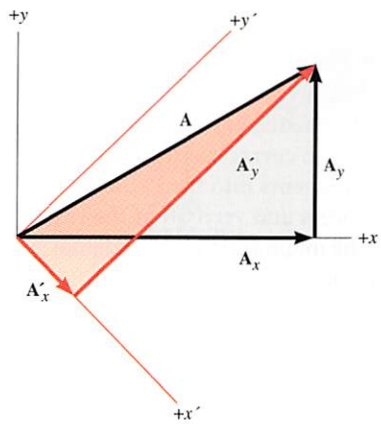


Figure 1.18 The vector components of the vector depend on the orientation of the axes used as a reference.

Scalar components

It is often easier to work with the ***scalar components,*** A_x and A_y (note the italic symbols), rather than the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$. Scalar components are positive or negative numbers (with units) that are defined as follows. The component A_x has a magnitude that is equal to that of $\mathbf{A}_x$ and is given a positive sign if $\mathbf{A}_x$ points along the $+x$ axis and a negative sign if $\mathbf{A}_x$ points along the $-x$ axis. The component A_y is defined in a similar manner. The following table shows an example of vector and scalar components:

Vector Components	Scalar Components
$\mathbf{A}_x$ = 8 meters, directed along the $+x$ axis	$A_x = +8$ meters
$\mathbf{A}_y$ = 10 meters, directed along the $-y$ axis	$A_y = -10$ meters

In this text, when we use the term "component," we will be referring to a scalar component, unless otherwise indicated.

Resolving a vector into its components

If the magnitude and direction of a vector are known, it is possible to find the components of the vector. The process of finding the components is called "resolving the vector into its components." As Example 7 illustrates, this process can be carried out with the aid of trigonometry, because the two perpendicular vector components and the original vector form a right triangle.

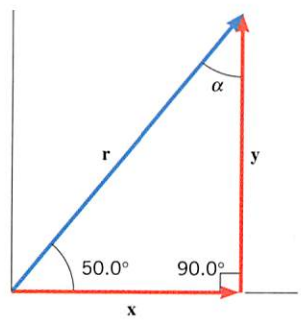


Figure 1.19 The x and y components of the displacement vector $\mathbf{r}$ can be found using trigonometry.

EXAMPLE 7 • Finding the Components of a Vector

A displacement vector $\mathbf{r}$ has a magnitude of $r = 175$ m and points at an angle of 50.0° relative to the x axis in Figure 1.19. Find the x and y components of this vector.

Reasoning and Solution 1 The y component can be obtained using the 50.0° angle and Equation 1.1, $\sin\theta = y/r$:

$$y = r\sin\theta = (175\text{ m})(\sin 50.0°) = \boxed{134\text{ m}}$$

In a similar fashion, the x component can be obtained using the 50.0° angle and Equation 1.2, $\cos\theta = x/r$:

$$x = r\cos\theta = (175\text{ m})(\cos 50.0°) = \boxed{112\text{ m}}$$

Reasoning and Solution 2 The angle α in Figure 1.19 can also be used to find the components. Since $\alpha + 50.0° = 90.0°$, it follows that $\alpha = 40.0°$. The solution using α yields the same answers as in Solution 1:

$$\cos\alpha = \frac{y}{r}$$

$$y = r\cos\alpha = (175\text{ m})(\cos 40.0°) = \boxed{134\text{ m}}$$

$$\sin\alpha = \frac{x}{r}$$

$$x = r\sin\alpha = (175\text{ m})(\sin 40.0°) = \boxed{112\text{ m}}$$

• Problem solving insight
Either acute angle of a right triangle can be used to determine the components of a vector. The choice of angle is a matter of convenience.

• PROBLEM SOLVING INSIGHT
When a vector is resolved into components, one can check to see if they are correct; substitute the components into the Pythagorean theorem and verify that the result is the magnitude of the original vector.

Since the vector components and the original vector form a right triangle, the Pythagorean theorem can be applied to check the validity of calculations such as those in Example 7. Thus, with the components obtained in Example 7, the theorem can be used to verify that the magnitude of the original vector is indeed 175 m, as given initially:

$$r = \sqrt{(112\text{ m})^2 + (134\text{ m})^2} = 175\text{ m}$$

VECTORS THAT HAVE ZERO COMPONENTS

Depending on the orientation of the axes used as a reference, it is possible that one of the components of a vector can be zero. Figure 1.20 shows an example of this situation to emphasize that a vector is not zero merely because one of its components is zero. In this drawing, the y vector component is itself the vector **A**, the x vector component being zero. Vector **A** would be expressed as the sum of its vector components according to the following vector equation: $\mathbf{A} = 0 + \mathbf{A}_y$.

For a vector to be zero, every vector component must individually be zero. Thus, in two dimensions, saying that $\mathbf{A} = 0$ is equivalent to saying that $\mathbf{A}_x = 0$ and $\mathbf{A}_y = 0$. Or, stated in terms of scalar components, if $\mathbf{A} = 0$, then $A_x = 0$ and $A_y = 0$. This seemingly trivial fact plays an important role in physics. In particular, it will be used in Chapter 4 when we describe the equilibrium of an object by saying that the net force acting on the object is zero.

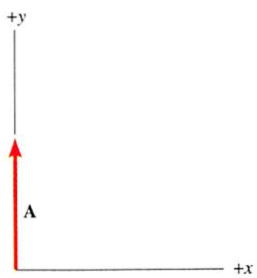


Figure 1.20 The x vector component of the vector **A** is zero, although the vector itself is not zero.

VECTORS THAT ARE EQUAL

Two vectors are equal if, and only if, they have the same magnitude and direction. Thus, if one displacement vector points east and another points north, they are *not* equal, even if each has the same magnitude of 480 m. In terms of vector components, two vectors, **A** and **B**, are equal if, and only if, each vector component of one is equal to the corresponding vector component of the other. In two dimensions, if $\mathbf{A} = \mathbf{B}$, then $\mathbf{A}_x = \mathbf{B}_x$ and $\mathbf{A}_y = \mathbf{B}_y$. Alternatively, using scalar components, we write that $A_x = B_x$ and $A_y = B_y$.

1.8 ADDITION OF VECTORS BY MEANS OF COMPONENTS

The components of a vector provide the most convenient and accurate way of adding (or subtracting) any number of vectors. For example, suppose that vector **A** is added to vector **B**. The resultant vector is **C**, where $\mathbf{C} = \mathbf{A} + \mathbf{B}$. Figure 1.21*a* illustrates this vector addition, along with the x and y vector components of **A** and **B**. In part *b* of the drawing, the vectors **A** and **B** have been removed, because we can use the vector components of these vectors in place of them. The vector component $\mathbf{B}_x$ has been shifted downward and arranged tail-to-head with the vector component $\mathbf{A}_x$. Similarly, the vector component $\mathbf{A}_y$ has been shifted to the right and arranged tail-to-head with the vector component $\mathbf{B}_y$. The x components are colinear and add together to give the x component of the resultant vector **C**. In like fashion, the y components are colinear and add together to give the y component of **C**. In terms of scalar components, we can write

$$C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y$$

The vector components $\mathbf{C}_x$ and $\mathbf{C}_y$ of the resultant vector form the sides of the right

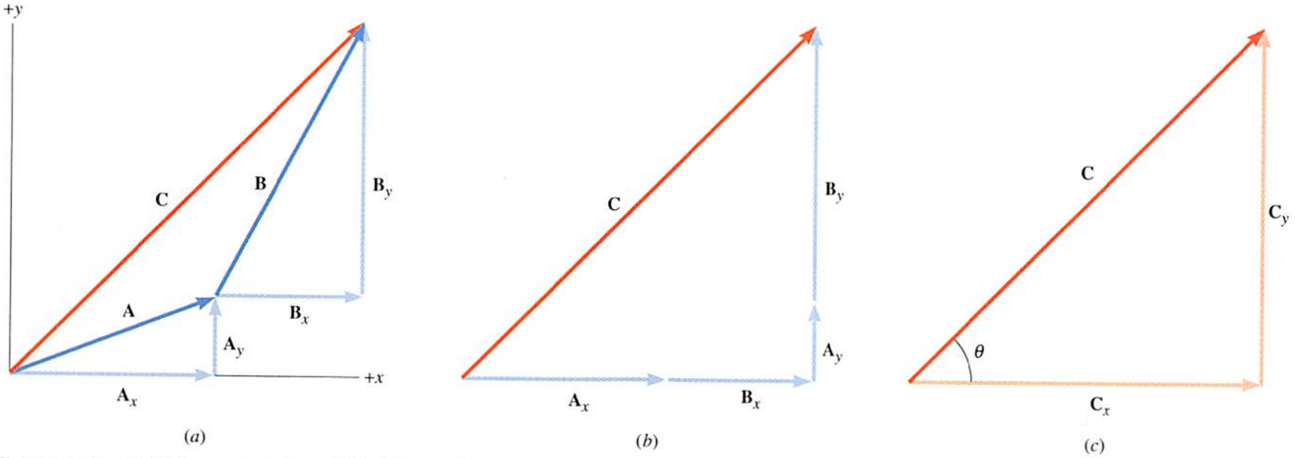


Figure 1.21 (*a*) The vectors **A** and **B** add together to give the resultant vector **C**. The x and y vector components of **A** and **B** are also shown. (*b*) The drawing illustrates that $\mathbf{C}_x = \mathbf{A}_x + \mathbf{B}_x$ and $\mathbf{C}_y = \mathbf{A}_y + \mathbf{B}_y$. (*c*) Vector **C** and its components form a right triangle.

triangle shown in Figure 1.21*c*. Thus, we can find the magnitude of **C** by using the Pythagorean theorem:

$$C = \sqrt{C_x^2 + C_y^2}$$

The angle θ that **C** makes with the x axis is given by $\theta = \tan^{-1}(C_y/C_x)$. Example 8 illustrates how to add several vectors using the component method.

EXAMPLE 8 • The Component Method of Vector Addition

A jogger runs 145 m in a direction 20.0° east of north (displacement vector **A**) and then 105 m in a direction 35.0° south of east (displacement vector **B**). Determine the magnitude and direction of the resultant vector **C** for these two displacements.

Reasoning Figure 1.22*a* shows the vectors **A** and **B**, assuming that the y axis corresponds to the direction due north. Since the vectors are not given in component form, we will begin by using the given magnitudes and directions to find the components. Then the components of **A** and **B** can be used to find the components of the resultant **C**. Finally, with the aid of the Pythagorean theorem and trigonometry, the components of **C** can be used to find its magnitude and direction.

Solution The first two rows of the table below give the x and y components of the vectors **A** and **B**. Note that the component B_y is negative, because $\mathbf{B}_y$ points downward, in the negative y direction in the drawing.

Vector	x component	y component
A	$A_x = (145\text{ m}) \sin 20.0° = 49.6\text{ m}$	$A_y = (145\text{ m}) \cos 20.0° = 136\text{ m}$
B	$B_x = (105\text{ m}) \cos 35.0° = 86.0\text{ m}$	$B_y = -(105\text{ m}) \sin 35.0° = -60.2\text{ m}$
C	$C_x = A_x + B_x = 135.6\text{ m}$	$C_y = A_y + B_y = 76\text{ m}$

The third row in the table gives the x and y components of the resultant vector **C**: $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Part *b* of the drawing shows **C** and its vector components. The magnitude of **C** is given by the Pythagorean theorem as

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(135.6\text{ m})^2 + (76\text{ m})^2} = \boxed{155\text{ m}}$$

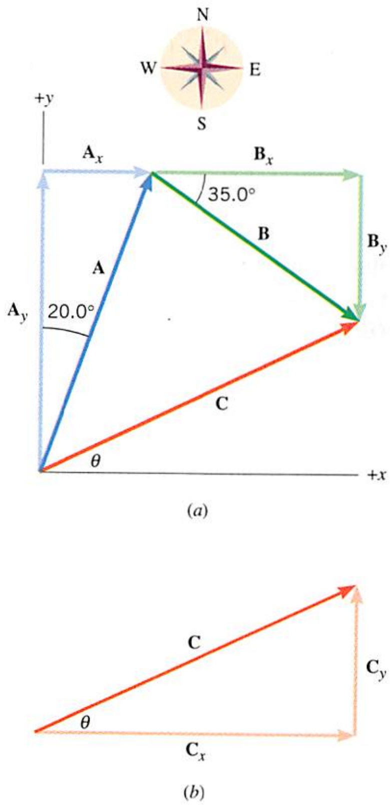


Figure 1.22 (*a*) The vectors **A** and **B** add together to give the resultant vector **C**. The vector components of **A** and **B** are also shown. (*b*) The resultant vector **C** can be obtained once its components have been found.

The angle θ that **C** makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{76\text{ m}}{135.6\text{ m}}\right) = \boxed{29°}$$

In later chapters we will often use the component method for vector addition. For future reference, the main features of the reasoning strategy used in this technique are summarized below.

REASONING STRATEGY

The Component Method of Vector Addition

1. For each vector to be added, determine the x and y components relative to a conveniently chosen x, y coordinate system. Be sure to take into account the directions of the components by using plus and minus signs to denote whether the components point along the positive or negative axes.
2. Find the algebraic sum of the x components, which is the x component of the resultant vector. Similarly, find the algebraic sum of the y components, which is the y component of the resultant vector.
3. Use the x and y components of the resultant vector and the Pythagorean theorem to determine the magnitude of the resultant vector.
4. Use either the inverse sine, inverse cosine, or inverse tangent function to find the angle that specifies the direction of the resultant vector.

SUMMARY

Physics is an experimental science that uses precisely defined **units of measurement.** This text emphasizes SI (Système International) units, a system that includes the meter (m), the kilogram (kg), and the second (s) as base units for length, mass, and time, respectively. Units play an important role in solving problems, because the units on the left side of an equation must match the units on the right side. If the units on both sides do not match, either the equation is written incorrectly or the variables and constants in the equation are not expressed in a consistent set of units.

Trigonometry is used throughout physics. Particularly important are the sine, cosine, and tangent functions of an angle θ. These functions can be defined in terms of a right triangle that contains θ. The side of the triangle opposite θ is h_o, the side adjacent to θ is h_a, and the hypotenuse is h. In terms of these quantities $\sin\theta = h_o/h$, $\cos\theta = h_a/h$, and $\tan\theta = h_o/h_a$. Once the value of the sine, cosine, or tangent is known, the angle itself can be obtained using inverse trigonometric functions. The Pythagorean theorem, $h^2 = h_o^2 + h_a^2$, is useful when dealing with the sides of a right triangle.

A **scalar quantity** is described completely by its size, which is also called its magnitude. For a **vector quantity,** however, both magnitude and direction must be specified. Vectors are often represented by arrows, the length of the arrow being proportional to the magnitude of the vector and the direction of the arrow indicating the direction of the vector. The **addition of vectors** to give a resultant vector must account for both magnitude and direction. When the vectors are all colinear, the addition proceeds in the same way as the simple addition of scalar quantities. When the vectors are not colinear, one procedure for addition utilizes a graphical technique, in which the vectors to be added are arranged in a tail-to-head fashion. The subtraction of a vector is treated as the addition of a vector that has been multiplied by a scalar factor of -1. Multiplying a vector by -1 reverses the direction of the vector.

In two dimensions, the **vector components** of a vector **A** are two perpendicular vectors $\mathbf{A}_x$ and $\mathbf{A}_y$ that are parallel to the x and y axes, respectively, and add together vectorially so that $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$. The **scalar component** A_x has a magnitude that is equal to that of $\mathbf{A}_x$ and is given a positive sign if $\mathbf{A}_x$ points along the $+x$ axis and a negative sign if $\mathbf{A}_x$ points along the $-x$ axis. The scalar component A_y is defined in a similar manner. Components provide the best way of adding any number of vectors. A vector is zero if, and only if, each of its vector components is zero. Two vectors are equal in two dimensions if, and only if, the x vector components of each are equal and the y vector components of each are equal.

PROBLEMS

Problems that are not marked with a star are considered the easiest to solve. Problems that are marked with a single star (*) are more difficult, while those marked with a double star (**) are the most difficult.

ssm Solution is in the Student Solutions Manual. www Solution is available on the World Wide Web at http://www.wiley.com/college/cutnell This icon represents a biomedical application.

Section 1.3 The Role of Units in Problem Solving

1. **ssm** The mass of the parasitic wasp *Caraphractus cintus* can be as small as 5×10^{-6} kg. What is this mass in (a) grams (g), (b) milligrams (mg), and (c) micrograms (μg)?

2. The distance of the Boston marathon is 26 miles, 385 yards. What is the length of this race in meters?

3. How many seconds are there in (a) one hour and thirty-five minutes and (b) one day?

4. A 747 jetliner is cruising at a speed of 520 miles per hour. What is its speed in kilometers per hour?

5. **ssm** The largest diamond ever found had a size of 3106 carats. One carat is equivalent to a mass of 0.200 g. Use the fact that 1 kg (1000 g) has a weight of 2.205 lb under certain conditions, and determine the weight of this diamond in pounds.

6. A bottle of wine known as a magnum contains a volume of 1.5 liters. A bottle known as a jeroboam contains 0.792 U.S. gallons. How many magnums are there in one jeroboam?

7. The following are dimensions of various physical parameters that will be discussed later on in the text. Here [L], [T], and [M] denote, respectively, dimensions of length, time, and mass.

	Dimension		Dimension
Distance (x)	[L]	Acceleration (a)	$[L]/[T]^2$
Time (t)	[T]	Force (F)	$[M][L]/[T]^2$
Mass (m)	[M]	Energy (E)	$[M][L]^2/[T]^2$
Speed (v)	[L]/[T]		

Which of the following equations are dimensionally correct?

(a) $F = ma$ (d) $E = max$

(b) $x = \frac{1}{2}at^3$ (e) $v = \sqrt{Fx/m}$

(c) $E = \frac{1}{2}mv$

8. The variables x, v, and a have the dimensions of [L], [L]/[T], and $[L]/[T]^2$, respectively. These variables are related by an equation that has the form $v^n = 2ax$, where n is an integer constant (1, 2, 3, etc.) without dimensions. What must be the value of n, so that both sides of the equation have the same dimensions? Explain your reasoning.

***9.** **ssm** The depth of the ocean is sometimes measured in fathoms (1 fathom = 6 feet). Distance on the surface of the ocean is sometimes measured in nautical miles (1 nautical mile = 6076 feet). The water beneath a surface rectangle 1.20 nautical miles by 2.60 nautical miles has a depth of 16.0 fathoms. Find the volume of water (in cubic meters) beneath this rectangle.

***10.** The CGS unit for measuring the viscosity of a liquid is the poise [P]: 1 P = 1 g/(s·cm). The SI unit is the kg/(s·m). The viscosity of water at 0 °C is 1.78×10^{-3} kg/(s·m). Express this viscosity in poise.

Section 1.4 Trigonometry

11. The gondola ski lift at Keystone, Colorado, is 2830 m long. On average, the ski lift rises 14.6° above the horizontal. How high is the top of the ski lift relative to the base?

12. A hill that has a 12.0% grade is one that rises 12.0 m vertically for every 100.0 m of distance in the horizontal direction. At what angle is such a hill inclined above the horizontal?

13. **ssm www** A highway is to be built between two towns, one of which lies 35.0 km south and 72.0 km west of the other. What is the shortest length of highway that can be built between the two towns, and at what angle would this highway be directed with respect to due west?

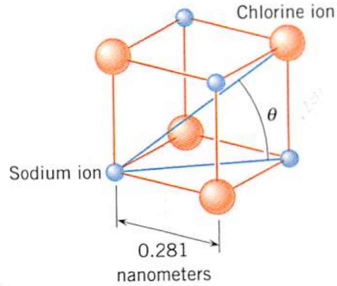
14. You are driving into St. Louis, Missouri, and in the distance you see the famous Gateway-to-the-West arch. This monument rises to a height of 192 m. You estimate your line of sight with the top of the arch to be 2.0° above the horizontal. Approximately how far (in kilometers) are you from the base of the arch?

15. The silhouette of a Christmas tree is an isosceles triangle. The angle at the top of the triangle is 30.0°, and the base measures 2.00 m across. How tall is the tree?

16. An observer, whose eyes are 1.83 m above the ground, is standing 32.0 m away from a tree. The ground is level, and the tree is growing perpendicular to it. The observer's line of sight with the treetop makes an angle of 20.0° above the horizontal. How tall is the tree?

***17.** **ssm** What is the value of each of the angles of a triangle whose sides are 95, 150, and 190 cm in length? *(Hint: Consider using the law of cosines given in Appendix E.)*

***18.** The drawing shows sodium and chlorine ions positioned at



the corners of a cube that is part of the crystal structure of sodium chloride (common table salt). The edge of the cube is 0.281 nm (1 nm = 1 nanometer = 10^{-9} m) in length. Find the distance (in nanometers) between the sodium ion located at one corner of the cube and the chlorine ion located on the diagonal at the opposite corner.

*19. What is the value of the angle θ in the drawing that accompanies problem 18?

**20. A regular tetrahedron is a three-dimensional object that has four faces, each of which is an equilateral triangle. Each of the edges of such an object has a length L. The height H of a regular tetrahedron is the perpendicular distance from one corner to the center of the opposite triangular face. Show that the ratio between H and L is $H/L = \sqrt{2/3}$.

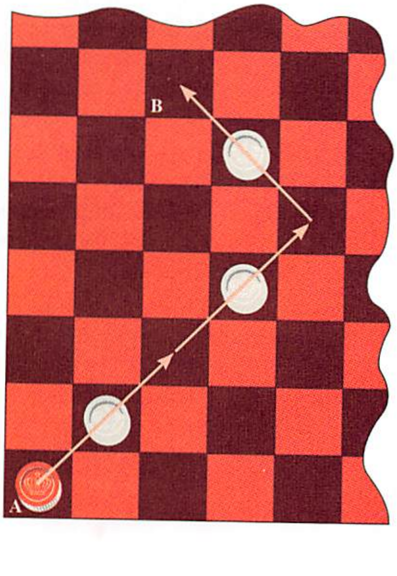
Section 1.6 Vector Addition and Subtraction

21. **ssm www** One displacement vector **A** has a magnitude of 2.43 km and points due north. A second displacement vector **B** has a magnitude of 7.74 km and also points due north. (a) Find the magnitude and direction of **A** − **B**. (b) Find the magnitude and direction of **B** − **A**.

22. A chimpanzee sitting against his favorite tree gets up and walks 51 m due east and 39 m due south to reach a termite mound, where he eats lunch. (a) What is the shortest distance between the tree and the termite mound? (b) What angle does the shortest distance make with respect to due east?

23. A force vector $\mathbf{F}_1$ points due east and has a magnitude of 200 newtons. A second force $\mathbf{F}_2$ is added to $\mathbf{F}_1$. The resultant of the two vectors has a magnitude of 400 newtons and points along the east/west line. Find the magnitude and direction of $\mathbf{F}_2$. Note that there are two answers.

24. The drawing shows a triple jump on a checkerboard, starting at the center of square A and ending on the center of square B. Each side of a square measures 4.0 cm. What is the magnitude of the displacement of the colored checker during the triple jump?



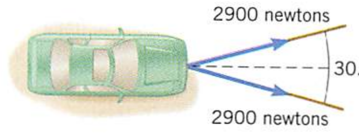
25. **ssm** Two ropes are attached to a heavy box to pull it along the floor. One rope applies a force of 475 newtons in a direction due west; the other applies a force of 315 newtons in a direction due south. As we will see later in the text, force is a vector quantity. (a) How much force should be applied by a single rope, and (b) in what direction (relative to due west), if it is to accomplish the same effect as the two forces added together?

26. A jogger travels due south, and in the process his displacement vector has a magnitude of 4.68 km. He then jogs due west. (a) What is the magnitude of his displacement vector in the due west direction, if the magnitude of his total displacement vector is 7.41 km? (b) What is the direction of his total displacement vector with respect to due south?

27. Vector **A** has a magnitude of 48.0 units and points due west, while vector **B** has the same magnitude but points due south. Determine the magnitude and direction of (a) **A** + **B** and (b) **A** − **B**. Specify the direction relative to due west.

*28. A basketball player runs a pattern consisting of three segments. The corresponding three displacement vectors **A**, **B**, and **C** have equal magnitudes of 7.0 m. Displacement **A** is directed forward and parallel to one side of the court, **B** is directed forward at a 45° angle with respect to the side of the court, and **C** is directed forward and parallel to the side of the court. With a scale drawing, use the graphical technique to find the magnitude and direction of the displacement vector for a straight-line dash between the starting and finishing points.

*29. **ssm www** A car is being pulled out of the mud by two forces that are applied by the two ropes shown in the drawing. The dashed line in the drawing bisects the 30.0° angle. The magnitude of the force applied by each rope is 2900 newtons. Arrange the force vectors tail to head and use the graphical technique to answer the following questions. (a) How much force would a single rope need to apply to accomplish the same effect as the two forces added together? (b) How would the single rope be directed relative to the dashed line?



*30. In wandering, a grizzly bear makes a displacement of 1563 m due west, followed by a displacement of 3348 m in a direction 32.0° north of west. What are (a) the magnitude and (b) the direction of the displacement needed for the bear to *return to its starting point?* Specify the direction relative to due east.

*31. Vector **A** has a magnitude of 8.00 units and points due west. Vector **B** points due north. (a) What is the magnitude of **B** if **A** + **B** has a magnitude of 10.00 units? (b) What is the direction of **A** + **B** relative to due west? (c) What is the magnitude of **B** if **A** − **B** has a magnitude of 10.00 units? (d) What is the direction of **A** − **B** relative to due west?

Section 1.7 The Components of a Vector

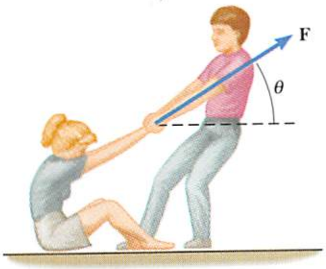
32. A displacement vector has a magnitude of 177 m and points at an angle of 36.0° below the positive x axis. What are (a) the x scalar component and (b) the y scalar component of the vector?

33. **ssm** Vector **A** points along the $+y$ axis and has a magnitude of 100.0 units. Vector **B** points at an angle of 60.0° above the $+x$ axis and has a magnitude of 200.0 units. Vector **C** points along the $+x$ axis and has a magnitude of 150.0 units. Which vector has (a) the largest x component and (b) the largest y component?

34. A bicyclist is headed due east. A 5.00-m/s wind is blowing partially into the rider's face and is coming from a direction that is 35.0° south of east. The speed and direction of the wind constitute a vector quantity known as the velocity. In effect, then, the rider must "pump" against a component of the wind's velocity vector. What is the magnitude of this component?

35. An ocean liner leaves New York City and travels 18.0° north of east for 155 km. How far east and how far north has it gone? In other words, what are the magnitudes of the components of the ship's displacement vector in the directions (a) due east and (b) due north?

36. Your friend has slipped and fallen. To help him up, you pull with a force **F**, as the drawing shows. The vertical component of this force is 130 newtons, while the horizontal component is 150 newtons. Find (a) the magnitude of **F** and (b) the angle θ.

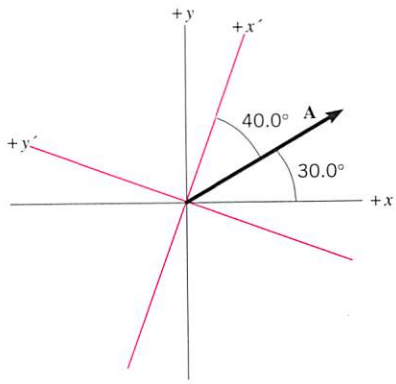


37. **ssm** The x vector component of a displacement vector **r** has a magnitude of 125 m and points along the negative x axis. The y vector component has a magnitude of 184 m and points along the negative y axis. Find the magnitude and direction of **r**. Specify the direction with respect to the negative x axis.

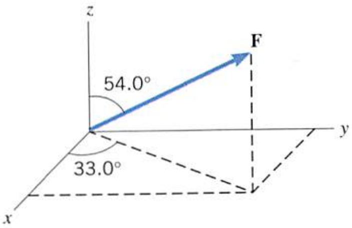
38. On takeoff, an airplane climbs with a speed of 180 m/s at an angle of 34° above the horizontal. The speed and direction of the airplane constitute a vector quantity known as the velocity. The sun is shining directly overhead. How fast is the shadow of the plane moving along the ground? (That is, what is the magnitude of the horizontal component of the plane's velocity?)

***39.** The magnitude of the force vector **F** is 280 newtons. The x component of this vector is directed along the $+x$ axis and has a magnitude of 150 newtons. The y component points along the $+y$ axis. (a) Find the direction of **F** relative to the $+x$ axis. (b) Find the component of **F** along the $+y$ axis.

***40.** The vector **A** in the drawing has a magnitude of 750 units. Determine the magnitude and direction of the x and y components of the vector **A**, relative to (a) the black axes and (b) the colored axes.



****41.** **ssm www** The drawing shows a force vector that has a magnitude of 475 newtons. Find the (a) x, (b) y, and (c) z components of the vector.



Section 1.8 Addition of Vectors by Means of Components

42. You are on a treasure hunt and your map says "Walk due west for 52 paces, then walk 30.0° north of west for 42 paces, and finally walk due north for 25 paces." What is the magnitude of the component of your displacement in the direction (a) due north and (b) due west?

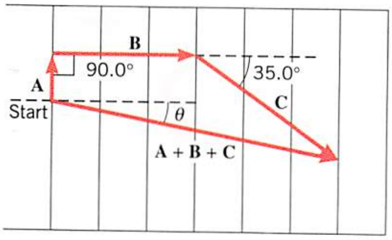
43. A pilot flies her route in two straight line segments. The displacement vector **A** for the first segment has a magnitude of 243 km and a direction 50.0° north of east. The displacement vector **B** for the second segment has a magnitude of 57.0 km and a direction 20.0° south of east. The resultant displacement vector is $\mathbf{R} = \mathbf{A} + \mathbf{B}$. What are the magnitude and direction of **R**? Use the component method and specify the direction relative to due east.

44. The force vector $\mathbf{F}_A$ has a magnitude of 45.0 newtons and points 30.0° north of east. The force vector $\mathbf{F}_B$ has a magnitude of 75.0 newtons and points due north. Find the magnitude and direction of the resultant $\mathbf{F}_A + \mathbf{F}_B$ by using the component method. Specify the direction relative to due east.

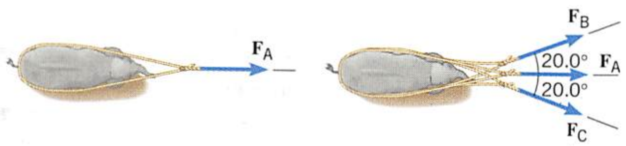
45. **ssm** A golfer, putting on a green, requires three strokes to "hole the ball." During the first putt, the ball rolls 5.0 m due east. For the second putt, the ball travels 2.1 m at an angle of 20.0° north of east. The third putt is 0.50 m due north. What displacement (magnitude and direction relative to due east) would have been needed to "hole the ball" on the very first putt?

46. On a safari, a team of naturalists sets out toward a research station located 4.8 km away in a direction 42° north of east. After traveling in a straight line for 2.4 km, they stop and discover that they have been traveling 22° north of east, because their guide misread his compass. What are (a) the magnitude and (b) the direction (relative to due east) of the displacement vector now required to bring the team to the research station?

47. A football player runs the pattern given in the drawing by the three displacement vectors **A**, **B**, and **C**. The magnitudes of these vectors are $A = 5.00$ m, $B = 15.0$ m, and $C = 18.0$ m. Using the component method, find the magnitude and direction θ of the resultant vector **A** + **B** + **C**.



48. A baby elephant is stuck in a mud hole. To help pull it out, game keepers use a rope to apply force $\mathbf{F}_A$, as part *a* of the drawing shows. By itself, however, force $\mathbf{F}_A$ is insufficient. Therefore, two additional forces $\mathbf{F}_B$ and $\mathbf{F}_C$ are applied, as in part *b* of the drawing. Each of these additional forces has the same magnitude *F*. The magnitude of the resultant force acting on the elephant in part *b* of the drawing is twice that in part *a*. Find the ratio F/F_A.



(*a*) (*b*)

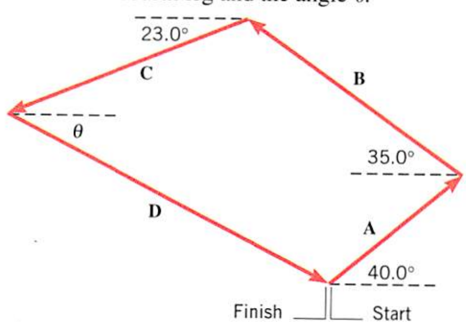
***49.** **ssm** Vector **A** has a magnitude of 6.00 units and points due east. Vector **B** points due north. (a) What is the magnitude of **B**, if the vector **A** + **B** points 60.0° north of east? (b) Find the magnitude of **A** + **B**.

***50.** Two forces act on an object. One has a magnitude of 166 newtons and points at an angle of 60.0° above the $+x$ axis. The second has a magnitude of 284 newtons and points at an angle of 30.0° above the $+x$ axis. A third force is applied and balances to zero the effects of the other two. What are the magnitude and direction of this third force? Specify the direction relative to the negative x axis.

***51.** Vector **A** has a magnitude of 188 units and points 30.0° north of west. Vector **B** points 50.0° east of north. Vector **C** points 20.0° west of south. These three vectors add to give a resultant vector that is zero. Using components, find the magnitudes of (a) vector **B** and (b) vector **C**.

***52.** A grasshopper makes four jumps. The displacement vectors are (1) 27.0 cm, due west; (2) 23.0 cm, 35.0° south of west; (3) 28.0 cm, 55.0° south of east; and (4) 35.0 cm, 63.0° north of east. Find the magnitude and direction of the resultant displacement. Express the direction with respect to due west.

***53.** **ssm** A sailboat race course consists of four legs, defined by the displacement vectors **A**, **B**, **C**, and **D**, as the drawing indicates. The magnitudes of the first three vectors are $A = 3.20$ km, $B = 5.10$ km, and $C = 4.80$ km. The finish line of the course coincides with the starting line. Using the data in the drawing, find the distance of the fourth leg and the angle θ.



ADDITIONAL PROBLEMS

54. The corners of an equilateral triangle lie on a circle that has a radius of 0.25 m. What is the length of a side of the triangle?

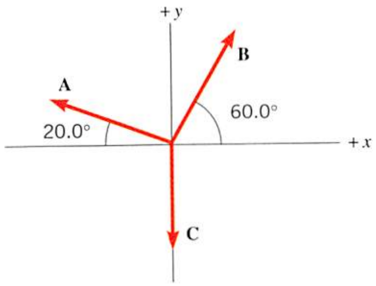
55. A displacement vector **A** has a magnitude of 1.62 km and points due north. Another displacement vector **B** has a magnitude of 2.48 km and points due east. Determine the magnitude and direction of (a) **A** + **B** and (b) **A** − **B**.

56. Consider the equation $v = \frac{1}{3}zxt^2$. The dimensions of the variables x, v, and t are [L], [L]/[T], and [T], respectively. What must be the dimensions of the variable z, such that both sides of the equation have the same dimensions? Show how you determined your answer.

57. **ssm** The speed of an object and the direction in which it moves constitute a vector quantity known as the velocity. An ostrich is running at a speed of 17.0 m/s in a direction of 68.0° north of west. What is the magnitude of the ostrich's velocity component that is directed (a) due north and (b) due west?

58. One acre contains 43 560 ft². How many square meters (m²) are in one acre?

59. Find the resultant of the three displacement vectors in the drawing by means of the component method. The magnitudes of the vectors are $A = 5.00$ m, $B = 5.00$ m, and $C = 4.00$ m.



60. A frog hops four times: twice forward, once to the right, and once forward again. Each hop covers a distance of 28 cm. What is the magnitude of the frog's displacement?

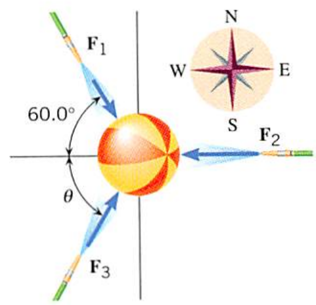
61. **ssm** Displacement vector **A** points due east and has a magnitude of 2.00 km. Displacement vector **B** points due north and has a magnitude of 3.75 km. Displacement vector **C** points due west and has a magnitude of 2.50 km. Displacement vector **D** points due south and has a magnitude of 3.00 km. Find the magnitude and direction (relative to due west) of the resultant vector **A + B + C + D**.

***62.** A displacement vector **A** has a magnitude of 636 m and points 40.0° above the $-x$ axis. Another displacement vector **B** is added to **A**. The resultant has the same magnitude as **A**, but the opposite direction. Find (a) the x component and (b) the y component of **B**.

***63.** Consider the two vectors **A** and **B** in the drawing for problem 59. Determine (a) the vector sum **A + B** and (b) the vector difference **A − B** using the method of components. In each case, specify both magnitude and direction (relative to the negative x axis).

***64.** Three deer, A, B, and C, are grazing in a field. Deer B is located 62 m from deer A at an angle of 51° north of west. Deer C is located 77° north of east relative to deer A. The distance between deer B and C is 95 m. What is the distance between deer A and C? *(Hint: Consider the law of cosines given in Appendix E.)*

***65.** At a picnic, there is a contest in which hoses are used to shoot water at a beach ball from three directions. As a result, three forces act on the ball, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ (see the drawing). The magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ are $F_1 = 50.0$ newtons and $F_2 = 90.0$ newtons. Using a scale drawing and the graphical technique, determine (a) the magnitude of $\mathbf{F}_3$ and (b) the angle θ such that the resultant force acting on the ball is zero.



***66.** Before starting this problem, review Conceptual Example 6. The force vector $\mathbf{F}_A$ has a magnitude of 90.0 newtons and points due east. The force vector $\mathbf{F}_B$ has a magnitude of 135 newtons and points 75° north of east. Use the graphical method and find the magnitude and direction of (a) $\mathbf{F}_A - \mathbf{F}_B$ (give the direction with respect to due east) and (b) $\mathbf{F}_B - \mathbf{F}_A$ (give the direction with respect to due west).

****67.** What are the x and y components of the vector that must be added to the following three vectors, so that the sum of the four vectors is zero? Due east is the $+x$ direction, and due north is the $+y$ direction.

A = 113 units, 60.0° south of west
B = 222 units, 35.0° south of east
C = 177 units, 23.0° north of east